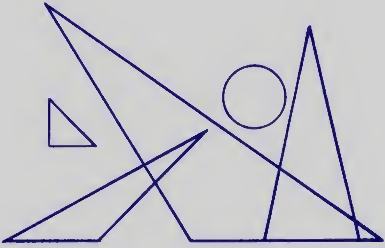
THINGS
of science



GEOMETRIC MODELS

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GEOMETRIC MODELS

Sometimes geometry students memorize theorems of geometry without understanding what they really mean. This series of models was developed for *THINGS of science* members with the cooperation of the National Council of Teachers of Mathematics to illustrate a variety of ideas and theorems from plane geometry.

The flexible models all deal with triangles, which are simply figures with three straight lines as sides.

Identify the various specimens you will be using to build your four models. The names of the models which you will construct are printed on the colored cardboard pieces to help you tell them apart.

TRIANGLES OF EQUAL AREA
MEDIAN OF A TRIANGLE
MIDPOINTS OF SIDES
CIRCLE AROUND TRIANGLE
PUSHPIN
THUMBTACK
SCREW EYE
ELASTIC THREAD

First cut your elastic into the following pieces:

One piece—6 inches long
One piece—5 inches long
Two pieces—5½ inches long
Two pieces—3½ inches long

The elastic thread for this unit was contributed by the TexElastic Corporation, High Point, N. C.

TRIANGLES OF EQUAL AREA

To make this model, using your pushpin make holes at A and B on the card marked "Triangles of Equal Area." Thread the 6-inch elastic through these holes, leaving a generous loop about 4 inches long on the front side of the model. Tie double knots on the reverse side so the elastic will not pull out. Your model is now ready for use.

Experiment 1. Loop your elastic around the pushpin and stick the pin into any point along the top line, well to the right of C. If the elastic is a bit too long to form a triangle with straight sides, shorten it from the back.

Does the three-sided figure made by the elastic and line AB seem to have about the same area as triangle ABC? (Fig. 1)

Experiment 2. Try sticking the pushpin at other points along the upper line.

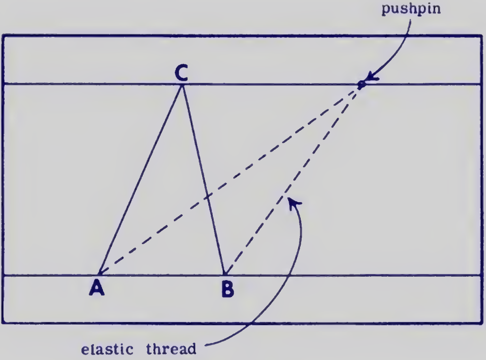

Fig. 1

Notice that when the pin is placed far to the right, the triangle is quite long and thin; when placed near C it becomes short and fat. In all cases the area of the triangle is equal to the area of ABC, the triangle already printed on the model. This is true because triangles with equal bases and equal altitudes are equal in area.

Experiment 3. Try varying the length of the sides to make the more familiar types of triangles (Fig. 2). These are:

Scalene triangle—no two sides equal in length.

Isosceles triangle—two sides equal.

Equilateral triangle—all three sides equal.

To make the first two triangles, the pushpin may be stuck in the upper line. For the equilateral triangle it should be located below the top parallel line and within the triangle ABC.

Experiment 4. Triangles may also be

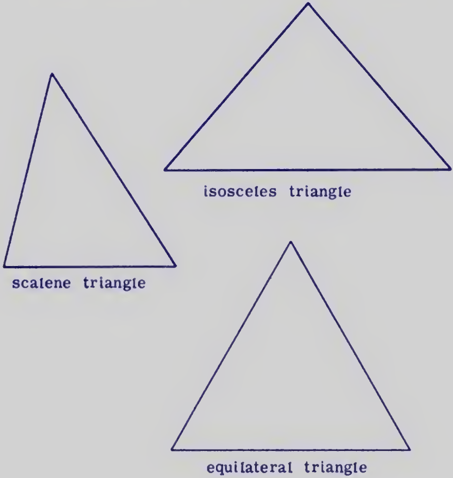

Fig. 2

classified as to the angles they contain:

Acute triangle—all three angles acute (smaller than a right angle which is a 90-degree angle made by two lines perpendicular to each other).

Equiangular triangle—all three angles equal.

Right triangle—one right angle.

Obtuse triangle—one obtuse angle (greater than a right angle).

If you insert your pin so the elastic goes straight up at A or B, you will have a right triangle. By moving the pin either to the right of the line perpendicular at B or to the left of the line perpendicular at A, you will form a triangle with two acute angles and one obtuse angle. If you place the pin between these two points, your triangle will have three acute angles.

Experiment 5. How would you make an equiangular triangle? All angles of an equilateral triangle are equal.

Experiment 6. An oblique triangle contains no right angle. Which of the triangles you have just made are oblique?

Experiment 7. Using your elastic and pushpin, how many triangles can you make of exactly the same shape and size as

triangle ABC? A triangle with sides and angles equal to the corresponding sides and angles of another triangle is said to be congruent to that triangle.

MEDIAN OF A TRIANGLE

To make your model, punch holes at A, B and M on the colored card marked "Median of a Triangle." Take a 5½-inch and a 3½-inch elastic thread.

Tie a double knot at one end of the longer elastic and thread it through at A from the back. Run the elastic through the screw eye, then thread it through B and knot it on the back side of the card. The portion of the elastic showing on the front of the model should be about 3½ inches long.

Now thread one end of the shorter piece of elastic through the screw eye and tie it securely to the screw eye. Insert the other end of the elastic through the hole at M and knot it, allowing about an inch and a quarter of elastic to show. Be sure the elastic is not drawn too taut when you insert the stem of the screw eye into the holes.

Experiment 8. Move the screw eye

from one to another of the holes punched into the cardboard and name the various types of triangles formed by the longer elastic and the line AB.

Experiment 9. Watch what the elastic from M (the midpoint between A and B) to the screw eye, which marks the vertex of the angle opposite the base, does to the triangle. Only in one position does it divide the large triangle into congruent triangles and only in one position does it bisect the vertex angle opposite the base. But it always divides the figure into two triangles of equal area (see Experiment 2).

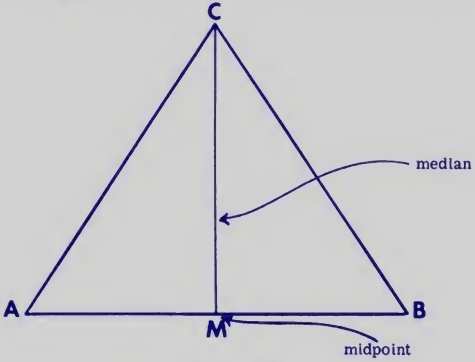

Fig. 3

Experiment 10. A median of a triangle is a line joining any vertex to the midpoint of the opposite side (Fig. 3).

Insert the screw eye into the hole at the far left or far right, and with a pencil lightly sketch another median of the triangle. Now draw a median connecting the third vertex with the midpoint of the opposite side. Notice that the three lines meet at a common point called the "center of gravity" of the triangle.

Experiment 11. Make an isosceles triangle (two sides equal) by inserting the screw eye into the appropriate hole. Notice that the median is now perpendicular to the base, and that the two triangles are congruent, each being the mirror image of the other. Lightly sketch in the other medians. Are they equal in length?

MIDPOINTS OF SIDES

Cut along the dotted line on the card marked "Midpoints of Sides." With the pushpin make holes at B and M on the card and C and M′ on the strip. Use a 3½-inch long and a 5-inch long piece of elastic thread for this model.

Thread the longer piece through B

and C knotting the ends at the back, and the shorter piece through M and M′ and knot. Insert the thumbtack through A′ and then A. To hold the thumbtack secure while doing your experiments, place your model on the cover of your THINGS box and push the thumbtack down into it.

Adjust the elastic pieces so that they are parallel when line A′M′C is perpendicular to the base BC.

Experiment 12. Rotate the strip to the right of line AMB and notice the relationship of the lines formed by the two elastics, one making the base of the triangle and the other connecting the midpoints of the other two sides. Use the push-pin to hold the strip at various positions and measure the perpendicular distance be-

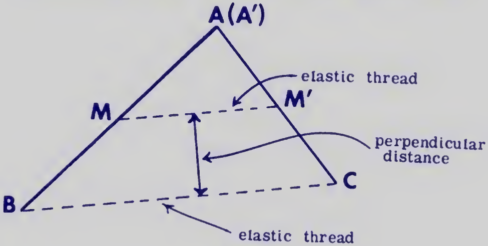

Fig. 4

tween the two elastics (Fig. 4). Are the lines always parallel?

This model illustrates the theorem: If a line joins the midpoints of two sides of a triangle, it is parallel to the third side. This is a special case of the theorem: If a line divides two sides of a triangle proportionally, it is parallel to the third side.

Experiment 13. Notice that the triangle formed by A (or A′), M and M′ is similar in shape to the triangle formed by A, B and C. Thus one method of creating similar triangles (where the corresponding angles are equal, but the corresponding sides are not necessarily equal in length) is to divide two sides of triangle proportionally and connect these two points. Another method obviously is to draw a line parallel to one of the sides.

CIRCUMSCRIBING A CIRCLE AROUND A TRIANGLE

Punch holes at each end of the chord (straight line connecting two points on the circumference of the circle) in your card marked "Circle around Triangle." Thread the 5½-inch piece of elastic through them and knot the ends at the back of the

model, leaving a loop about $3\frac{1}{4}$ inches long on the front.

Experiment 14. Loop the elastic around the pushpin and stick the pin into the card at various points around the circle, noting when you form acute, obtuse, right and isosceles triangles. Where does the center of the circle circumscribed around the various triangles lie in each case?

Experiment 15. Draw acute, obtuse, right and isosceles triangles on a piece of paper and try to guess where to locate the center of a circle that would pass through the three vertices. Now actually draw the circle you visualized. Did you guess correctly?

If you do not have a compass, make a loop of thread as long as the radius of your proposed circle. Slip a pencil through one end of the loop and through the other stick the pushpin, placing it in the spot chosen for locating the center of the circle. Draw an arc, holding the string between pencil and pushpin taut.

Experiment 16. On a piece of paper draw a large triangle. At the midpoint of each side, erect a line perpendicular to

that side. Use the point where these three lines cross as the center of a circle. When the circle passes through one of the vertices of the triangle, does it pass through them all?

Experiment 17. Take your model and form a right triangle by pushing the pin into the appropriate point on the circle. Note that one side of the triangle cuts directly across the center of the circle and thus is actually a diameter of the circle

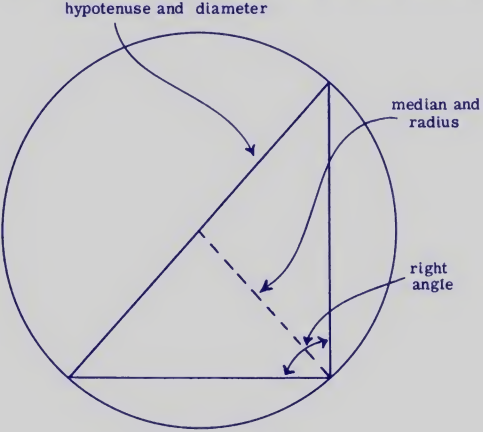

Fig. 5

with the center of that circle at its midpoint. This line is also the hypotenuse of the triangle, for by definition the hypotenuse is the side opposite the right angle. Draw the median from the vertex of the right angle to the hypotenuse. It is a radius of the circle and thus half as long as the hypotenuse (Fig. 5).

Experiment 18. Hold the pushpin so that both of the elastic sides of the triangle lie on the same side of the center of the circle. Notice that one of the angles of the triangle is greater than 90 degrees. Form other such triangles to demonstrate that all triangles inscribed in an arc less than a semicircle are obtuse.

MAKE LARGER MODELS

In case you wish to demonstrate principles of geometry to a group of students or friends, large permanent models may be made based upon these small ones. A construction board known as tempered hardboard makes the best base for your demonstration model, but wallboard, plywood, linoleum or extra heavy cardboard may be used. Black or white narrow elastic available in the notions department of

stores has enough snap to serve as the triangle sides.

Experiment 19. For the "Triangles of Equal Area" model, use a rectangular piece of hardboard 12 by 20 inches in size. Paint it red, orange or some other color. If hardboard is not available, cut one of the other materials listed to the appropriate size and cover one side with colored cardboard. Glue together and tape along the four edges.

Paint or draw two parallel lines 7½ inches apart across the entire length of the board, using a color that will contrast nicely with the background. Paint a triangle with an 8-inch base, using a part of the lower of the two parallel lines as the base, and with the vertex lying on the upper line. Locate the triangle slightly to the left of center. At the end points of the base of the triangle bore small holes through which you can lace the elastic. A nail or screw will help make these holes if you do not have better tools. Thread a piece of elastic about 22 inches long through to the back side and knot the ends securely.

If you used hardboard for this model,

at intervals along the upper line drive in small, short, round-headed screws. These screws should project 1/16 to 1/8 inch above the face of the model so that the elastic can be looped over them. Screws will not hold firmly in cardboard, so use long pointed pushpins should you use this material.

The model is now ready to demonstrate that triangles with equal bases and equal altitudes are always equal in area.

Experiment 20. To make your model of "Median of a Triangle" take a rectangular piece of hardboard 12 x 20 inches as before. Paint the hardboard as before in any desired color. About 2½ inches from the longer edge and parallel to it draw a black base line about a foot long. Locate and mark with a pencil the vertices of two obtuse triangles, two acute triangles, a right triangle and an isosceles triangle, all of which would have the 12-inch line as their base.

Find the midpoint of the base line. At the end points and at the midpoint, bore holes just large enough to take the elastic.

Obtain a large screw eye and a length of white elastic. Tie one end of the white

elastic to the screw eye and thread the other end through the hole at the midpoint of the base line. The points located for the vertices of the various triangles will determine how long the elastic should be.

Thread a long black elastic through the eye of the screw, then lace each end through a hole at the end point of the base line and tie at the back of the model. Test to be certain the points marking the vertices of the angles are well-placed, then at these points bore holes just large enough for the stem of the screw eye to fit snugly into them.

Experiment 21. Cut a rectangle of the same size for the "Midpoints of Sides" model. Paint it as before and draw a black line about 10 inches long to the left side of the rectangle, slanting it somewhat like the line in your smaller model.

Bore small holes at the ends and the midpoint of the line. Obtain a quarter inch dowel about 9 inches long and paint it the same color as the line on the rectangular piece if you wish. Bore a small hole about one inch from one end. Mark a point one inch from the other end on

the dowel. Make a similar mark at the midpoint.

Insert a small bolt through the hole at the end of the dowel and into the hole in the board at the upper end of the line. Then secure the bolt with a nut at the back of the model. Test to be sure the dowel turns easily.

Thread black elastic through the midpoint of the line and then tie it to the midpoint of the dowel, making a small but secure knot. Thread another longer elastic through the hole at the lower end of the line and tie the free end to the marked point on the dowel one inch from the end. Tie it securely. This elastic forms the base of the triangle.

Adjust the two elastic pieces by shortening or lengthening them from the back of the model and then knot the ends.

You are now ready to make all kinds of triangles by holding the dowel in various positions.

Experiment 22. A piece of hardboard 14 inches square will be fine for this demonstration model, "Circle around Triangle." Locate the center of the board and mark it carefully. Using this point

as the center, draw a circle 6 inches in radius. Paint the board in any color desired.

Draw or paint in black a chord about 8½ inches long and drill holes at the two end points. Thread elastic through these and knot on the back, leaving a loop of elastic about 17 inches long to work with. In the hardboard drive small, short, round-headed screws along the circle to locate the vertices of several acute triangles, obtuse triangles, two right triangles and an isosceles triangle. The screws should project 1/16 or ⅛ inch above the face of the model so the elastic can be looped over them.

Your model is now ready for experiments such as those suggested for the smaller model.

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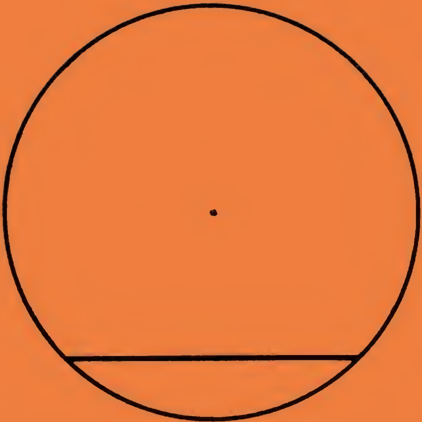
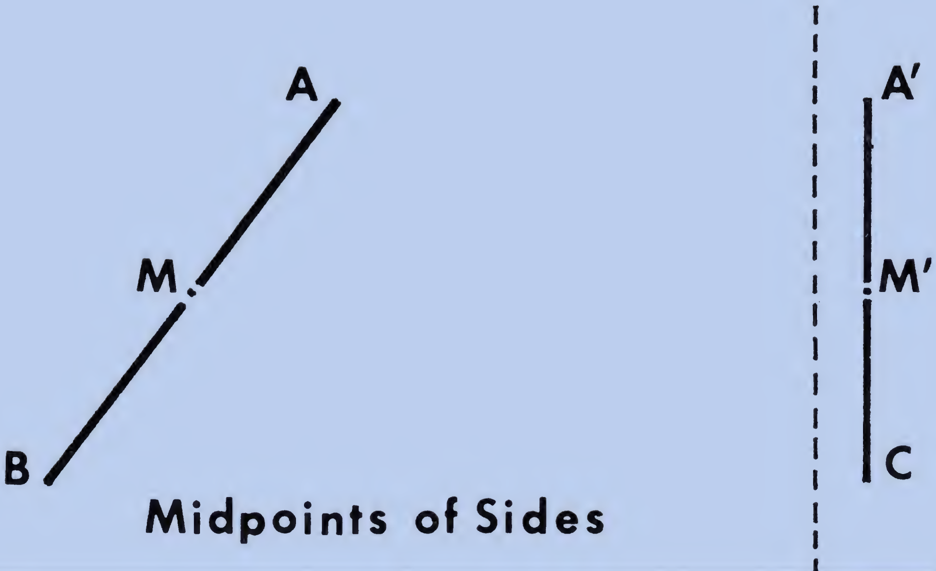
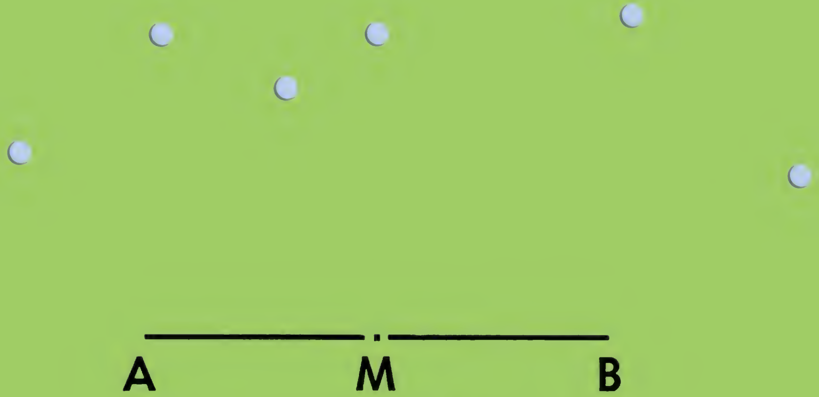


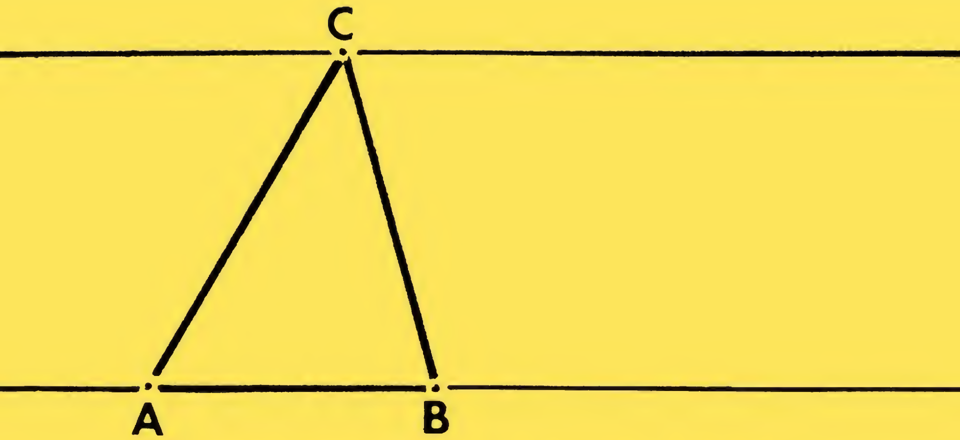
Circle around Triangle



Midpoints of Sides



Median of a Triangle



Triangles of Equal Area